# Introductory Finite Elements (3 units)

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## **Chapter 1.** Fundamental concepts

Stress and equilibrium; Principle of virtual work for solid mechanics problems; Principle of minimum potential energy; Galerkin-weighted residual and variational approaches; Rayleigh-Ritz method for solid mechanics problems

### Chapter 2. The concepts of finite element

Displacement based FE formulation; Strain-displacement and stress-strain relationships; The concepts of shape functions; Family of *C*-continuity ( $C_{0^{-}}$  and  $C_{1}$ -continuities); One-dimensional finite element; Stiffness matrix and force vectors for truss and beam elements; Lagrangian and Hermitian polynomial shape functions; Assembly of global stiffness matrix and force vectors

#### **Chapter 3.** Finite elements for two-dimensional problems

The shape functions of triangular elements (linear and higher orders); The shape functions of rectangular elements (linear and higher orders); Stiffness matrix for plane stress/strain problems (triangular and rectangular elements);

#### **Chapter 4.** Finite elements for three-dimensional problems

The shape functions of tetrahedral elements (linear and higher orders); The shape functions of brick elements (linear and higher orders); Stiffness matrix for three-dimensional problems (tetrahedral and brick elements)

#### Chapter 5. Isoparametric elements and Numerical integration

Natural coordinates; Area coordinates; Volume coordinates; Serendipity elements; Curved and isoparametric elements; Jacobian matrix for isoparametric elements; Numerical integration for 1D, 2D and 3D problems; Newton-Cotes approach, Gauss integration points

#### **Chapter 6.** Contact and interface elements

Stress-strain relationships; Interface element formulation; Shape functions and stiffness matrix of 2D and 3D interface elements

#### Chapter 7. Infinite domains and infinite elements

Shape functions and mapping functions; infinite line and infinite element map; Quadratic interpolations

#### Chapter 8. Automatic mesh generation

'Structured' and 'unstructured' mesh generators; Advancing front method; Delaunay method; Triangular and quadrilateral mesh generators

#### **References.**

- 1. D. J. Dawe, Matrix and Finite Element Displacement Analysis of Structures, *Oxford University Press*, New York, 1984.
- 2. O.C. Zienkiewicz and R.L. Taylor, The Finite Element Method, McGraw-Hill, London, Vol 1, 2000.

## **Evaluation.**

Final exam 50%, Project 30%, H.W. 20%